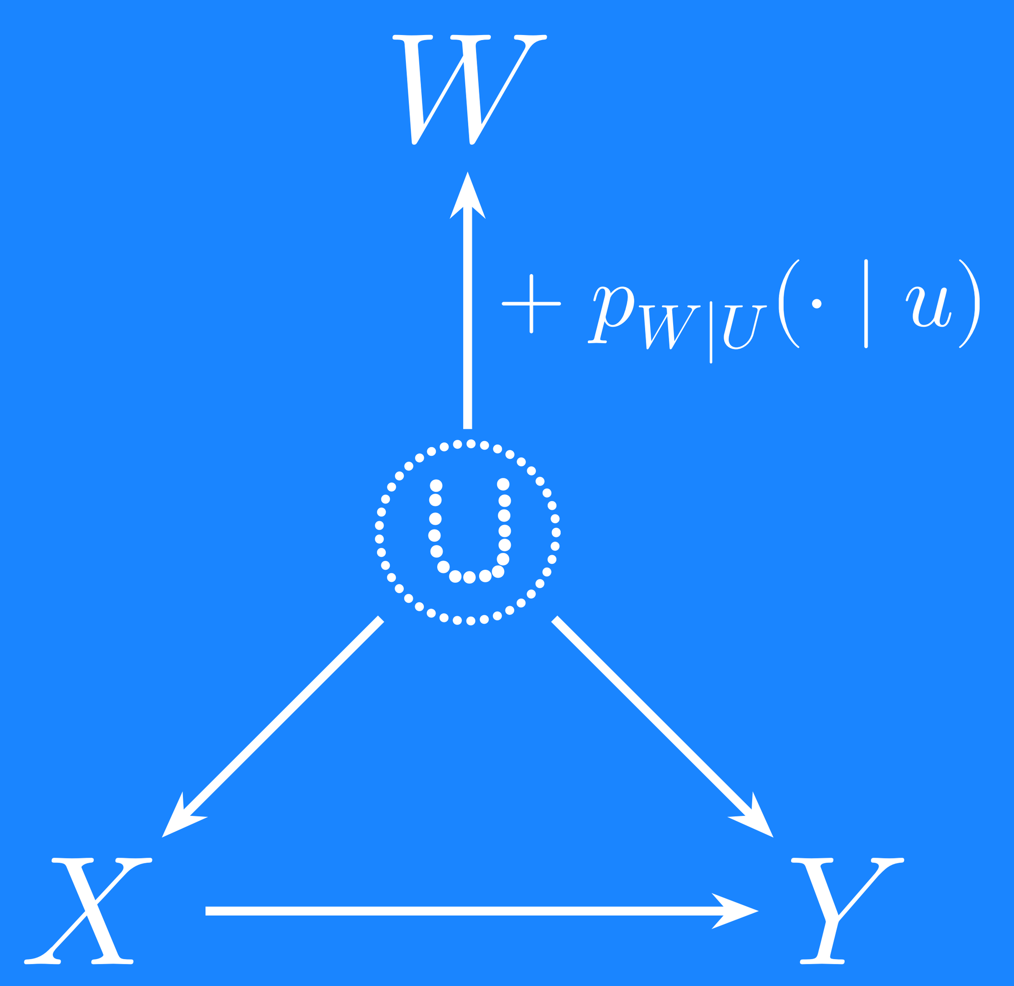
# We identify causal effects

**under** unobserved confounding **with**

**a known** *error mechanism*.




# Single Proxy Identifiability of Causal Effects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## structural causal model

$U \leftarrow N_U$
$W \leftarrow AU + E$
$X \leftarrow f_X(U, N_X)$
$Y \leftarrow f_Y(U, X, N_Y)$

- The proxy $W \in \mathcal{W} \subseteq \mathbb{R}^d$ has at least the dimension of the confounder $U \in \mathcal{U} \subseteq \mathbb{R}^k$, that is, $d \geq k$.
- $U \perp\!\!\!\perp E$ and $A \in \mathbb{R}^{d \times k}$ has full column rank.
- The Fourier transform of the density of $E$ has no zeros.

## causal function

Let $M_0$ be the true SCM. We define the causal function $\theta^{M_0} : \mathcal{X} \to \mathcal{Y}$ for all $x \in \mathcal{X}$ as

$$\theta^{M_0}(x) := \mathbb{E}_Y^{M_0;do(X:=x)}[Y] = \int y \int p^{M_0}_{Y|U,X}(y \mid u, x)\, p^{M_0}_U(u)\, \mu(du, dy).$$

## characteristic error mechanism

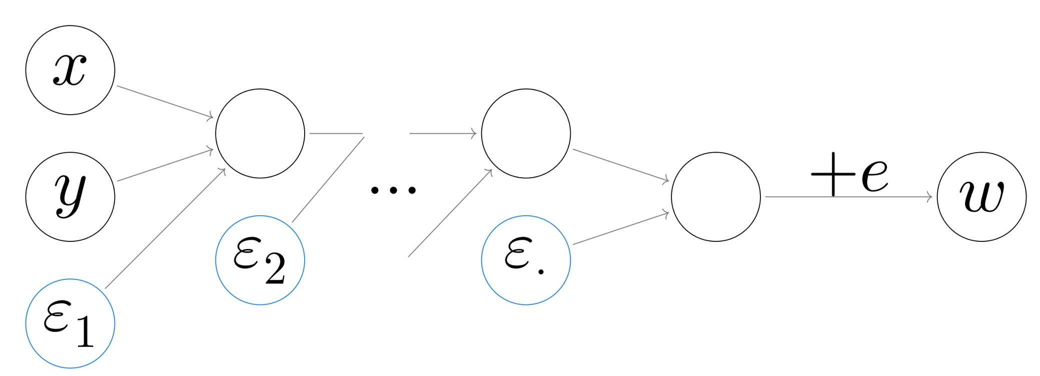
We show that the error mechanism $\{p^{M_0}_{W|U}(\cdot \mid u) \mid u \in \mathcal{U}\}$ is $L_\infty$-characteristic, that is, for all $\delta \in L_\infty(\mathcal{U})$

$$\int_{\mathcal{U}} p^{M_0}_{W|U}(\cdot \mid u)\delta(u)\, \mu(du) \equiv 0 \implies \delta \equiv 0 \quad \text{a.e.}$$

## identifiability of the causal function

If the error mechanism is $L_\infty$-characteristic and if for all SCMs, which are observationally equivalent to $M_0$, it holds that $p^{M}_{U|X,Y}(\cdot \mid x, y) \in L_\infty$ $\implies$ $\theta^{M_0}$ is identifiable.

## learn the causal function




1. Learn $P^{M_0}_{\widetilde{A}U|(X,Y)}$ by minimizing the Energy loss as in Engression [Shen and Meinshausen, 2023].
2. Adjust for $\widetilde{A}u$ with your favourite causal function estimator, e.g. EconML [Battocchi et al., 2019].